SP-07

4 (a) Prove that $n x(n) \xleftrightarrow{z} -z \frac{dX(z)}{dx}$.

where, $x(n) \xleftrightarrow{z} X(z)$

$$n x(n) \xleftrightarrow{z} -z \frac{dX(z)}{dz} \quad \text{--- (1)}$$

Now, Differentiating equ(1)

$$\frac{dX(z)}{dz} = \frac{d}{dz}\left\{\sum_{n=-\infty}^{\infty} x(n) z^{-n}\right\}$$

$$= \sum_{n=-\infty}^{\infty} x(n)(-n) z^{-n-1}$$

$$= -z^{-1} \sum_{n=-\infty}^{\infty} n x(n) \cdot z^{-n}$$

$$= -1/z \; Z\{n x(n)\}$$

$$\therefore -z \frac{dX(z)}{dz} \xleftrightarrow{z} n x(n)$$

SP-06

4(a) $(1/2)^n [u(n) - u(n-10)] = x(n)$

$$= \left(\frac{1}{2}\right)^n u(n) - \left(\frac{1}{2}\right)^n u(n-10)$$

$$\therefore X(z) = \frac{1}{1-\left(\frac{1}{2}\right)z^{-1}} - \left(\frac{1}{2}\right)^{10} \cdot (1/2)^{n-10} u(n-10)$$

$$= \frac{1}{1-(1/2)z^{-1}} - \left(\frac{1}{2}\right)^{10} \cdot z^{-10} \cdot \frac{1}{1-1/2 z^{-1}}$$

$$= \frac{1}{1-(1/2)z^{-1}} - \left(\frac{1}{2}\right)^{10} \cdot \frac{z^{-10}}{1-1/2 \cdot z^{-1}}$$

$$= \frac{1-(1/2)^{10} \cdot z^{-10}}{1-1/2 \cdot z^{-1}}$$

$$= \frac{1-\left(\frac{1}{2} \cdot z\right)^{10}}{1-1/2 \cdot z^{-1}}$$

Now pole and zero,

$$X(z) = \frac{\left\{1-(1/2 z^{-1})^{10}\right\} z^{10}}{(1-1/2 z^{-1}) z^{10}}$$

$$= \frac{z^{10} - (1/2)^{10}}{z^{10} - \frac{1}{2} z^{9}}$$

$$= \frac{z^{10} - (1/2)^{10}}{z^{9}(z-1/2)} \quad \text{Ans.}$$

F-06

Determine the response of the relaxed system characterized by the impulse response $h(n) = \left(\frac{1}{2}\right)^n u(n)$ to the input signal, ~~$x(n) = u(n) - u(n-10)$~~.

$$x(n) = \begin{cases} 1 & , \quad 0 \le n \le 10 \\ 0 & \quad \text{otherwise.} \end{cases}$$

Ans:- Now, $x(n) = u(n) - u(n-10)$

$$h(n) = (1/2)^n \, u(n)$$

$$u(n) * h(n) = \sum_{n=\alpha}^{\alpha} (1/2)^n \cdot u(n)$$

$$= \sum_{K=0}^{n} \left(\frac{1}{2}\right)^{n-K} u(K)$$

$$= (1/2)^n \sum_{K=0}^{n} 2^K.$$

$$= (1/2)^n \frac{2^{n+1} - 1}{2-1}$$

$$= (1/2)^n (2^{n+1} - 1)$$

$$= \left\{2 - (1/2)^n\right\}$$

$$\therefore \quad h(n) * u(n-10) = \left\{2 - \left(\frac{1}{2}\right)^{10}\right\}$$

$$\therefore Y(n) = h(n) * x(n) = \left\{u(n) - u(n-10)\right\} h(n)$$

$$= u(n)\,h(n) - u(n-10)\,h(n)$$

$$= \left\{2 - (1/2)^n\right\} - \left\{2 - (1/2)^{10}\right\} \quad \text{A}$$

S-06 fourier transformation.

5(a). $x(n) = (1/4)^n u(n+4)$

$$X(\omega) = \sum_{n=-4}^{\infty} (1/4)^n e^{-j\omega n}$$

$$= \sum_{m=0}^{\infty} \left(\frac{1}{4}\right)^{m-4} \left(e^{-j\omega}\right)^{m-4}$$

$\because m = n+4$
$m-4 = n.$
when $n=-4, m=0$
$n=\infty, m=\infty$

$$= \left(\frac{1}{4}\right)^{-4} \sum_{m=0}^{\infty} \left(\frac{1}{4}\right)^m e^{-j\omega m} e^{j\omega 4}$$

$$= 4^4 e^{j\omega 4} \sum_{m=0}^{\infty} \left(\frac{1}{4}\right)^m e^{-j\omega m}$$

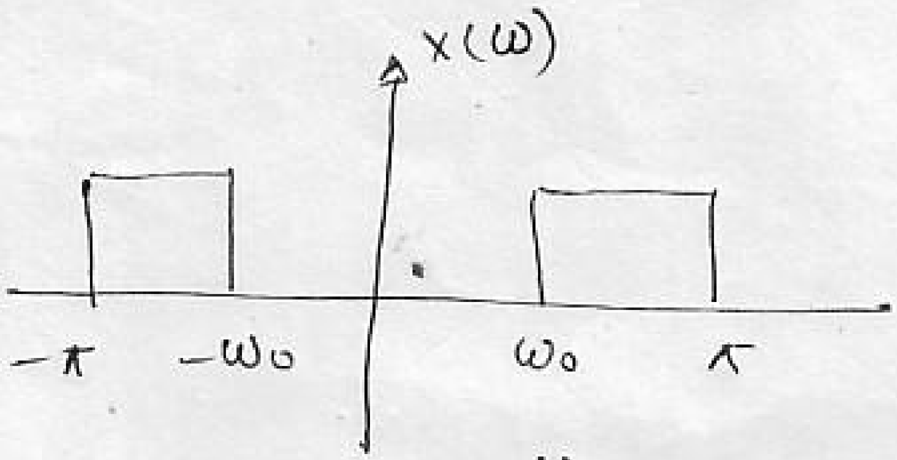
$$= 4^4 e^{j\omega 4} \sum_{m=0}^{\infty} \left(\frac{e^{-j\omega}}{4}\right)^m$$

$$= 4^4 e^{j\omega 4} \frac{1}{1 - \frac{e^{-j\omega}}{4}}$$

$$= \frac{4^4 e^{j\omega 4}}{1 - \frac{1}{4} e^{-j\omega}}$$

S-07 Fourier.

5(b) $X(\omega) = \begin{cases} 1 & \omega_0 \le |\omega| \le \pi \\ 0 & 0 \le |\omega| \le \omega_0 \end{cases}$



$$\therefore\ x(n) = \frac{1}{2\pi}\int_{-\pi}^{-\omega_0} e^{j\omega n}\,d\omega + \frac{1}{2\pi}\int_{\omega_0}^{\pi} e^{j\omega n}\,d\omega \quad (1)$$

$$= \frac{1}{2\pi}\left[\frac{e^{j\omega n}}{n}\right]_{-\pi}^{-\omega_0} + \frac{1}{2\pi}\left[\frac{e^{j\omega n}}{n}\right]_{\omega_0}^{\pi}$$

$$= \frac{1}{2\pi n}\left[e^{-j\omega_0 n} - e^{-j\pi n}\right] + \frac{1}{2\pi n}\left[e^{j\pi n} - e^{j\omega_0 n}\right]$$

$$= \frac{1}{2\pi j n}\left[e^{-j\omega_0 n} + e^{j\omega_0 n}\right]$$

$$= -\frac{1}{2\pi j n}\left[e^{j\omega_0 n} - e^{-j\omega_0 n}\right]$$

$$= -\frac{1}{\pi n}\sin\omega_0 n$$

$$= \frac{-\sin\omega_0 n}{\pi n}$$

Put, $n = 0$
on equ (1)

$$x(0) = \frac{1}{2\pi}\int_{-\pi}^{-\omega_0} d\omega + \frac{1}{2\pi}\int_{\omega_0}^{\pi} d\omega$$

$$= \frac{1}{2\pi}\left[\omega\right]_{-\pi}^{-\omega_0} + \frac{1}{2\pi}\left[\omega\right]_{\omega_0}^{\pi}$$

$$= \frac{1}{2\pi}\left(-\omega_0+\pi\right) + \frac{1}{2\pi}\left(\pi-\omega_0\right)$$

$$= \frac{1}{2\pi}\left(2\pi-2\omega_0\right)$$

$$x(0) = \frac{\pi-\omega_0}{\pi} \quad Ans$$