

(a) (i) Multichannel signal:

Sensor is a device which converts any physical quantity to an equivalent electrical signal.

The signal generated from multiple sources on sensor is called multichannel signal.

example: Electro cardiography signal (ECG)

(ii) multidimension signal:

If the signal is a quantity with more than one independent variable then it is called multidimensional signal.

Ex: Television signal.

(iii) Baseband form of sampling theorem:

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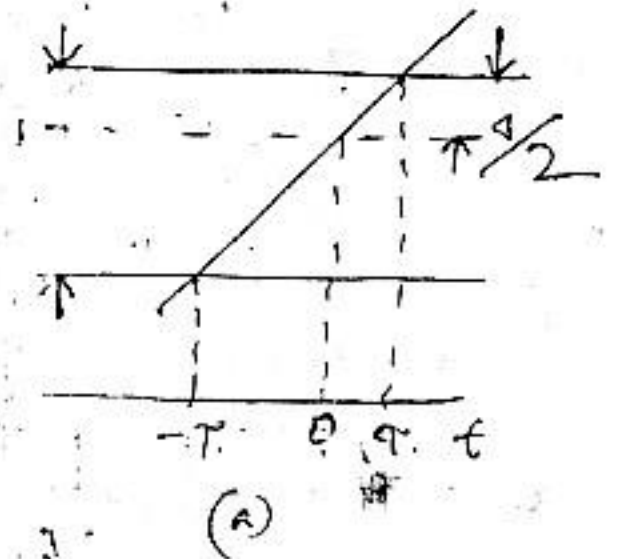
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(b) For a quantized sinusoid show that the SQNR increases approximately 6dB for every bit added to the word length.

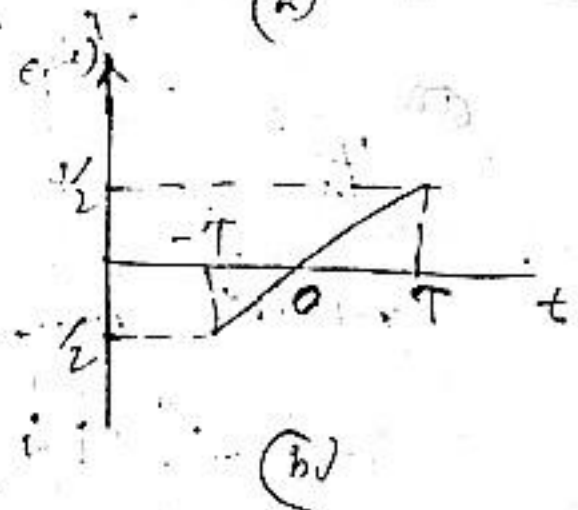
Ans: The quantization error e_q in the fig. (a) is

$$e_q(t) = x_d(t) - x_a(t)$$

$$e_q(t) = \frac{4V}{2T} t \quad \text{--- (1)}$$



in fig (b) T denotes the time that $x_d(t)$ stays within the quantization levels.



The mean-square error power P_{q_i} is

$$P_{q_i} = \frac{1}{2T} \int_{-T}^T e_q^2(t) dt$$

$$= \frac{1}{T} \int_0^T \left(\frac{4V}{2T} t \right)^2 dt$$

$$= \frac{4V^2}{4T^3} \frac{t^3}{3} \Big|_0^T = \frac{4V^2}{4T^3} \cdot \frac{1}{3} [T^3 - 0]$$

$$P_{q_i} = \frac{4V^2}{12} \quad \text{--- (2)}$$

If the quantizer has b bit of accuracy and the quantizer covers the entire range of the quantization

Summary

$$\text{So, } P_a = \frac{\left(\frac{2A}{2}\right)^2}{12} = \frac{4A^2}{2 \cdot 12}$$

$$= \frac{A^2/3}{2}$$

The average power of the signal $x_a(t)$ is

$$P_{av} = \frac{1}{T_p} \int_0^{T_p} (A \cos \omega t)^2 dt$$

$$= \frac{A^2}{T_p} \int_0^{T_p} \cos^2 \omega t dt$$

$$= \frac{A^2}{2T_p} \int_0^{T_p} (1 + \cos 2\omega t) dt$$

$$= \frac{A^2}{2T_p} \left[t + \frac{1}{2\omega} \sin 2\omega t \right]_0^{T_p}$$

$$= \frac{A^2}{2T_p} \left[\left[\frac{T_p}{1} - 0 \right] + \frac{1}{2\omega} \left[\sin 2\omega T_p - \sin 0 \right] \right]$$

$$P_{av} = \frac{A^2}{2T_p} \times T_p = \frac{A^2}{2}$$

The o/p of the A/D converter is usually measured by the signal-to-quantization noise ratio (SQNR), which provides the ratio of the signal power to the noise power.

$$\text{SQNR} = \frac{P_n}{P_q} = \frac{A^2}{2} \times \frac{3 \cdot 2^{2b}}{A^2}$$

$$= \frac{3}{2} \cdot 2^{2b}$$

Expressed in decibels (dB), the SQNR is

$$\begin{aligned} \text{SQNR (dB)} &= 10 \log_{10}(\text{SQNR}) \\ &= 10 \log_{10} \left(\frac{3}{2} \cdot 2^{2b} \right) \\ &= 10 \log_{10} \frac{3}{2} + 10 \log_{10} 2^{2b} \\ &= 1.76 + 2b \log_{10} 2 \end{aligned}$$

$$\boxed{\text{SQNR (dB)} = 1.76 + 6.02b}$$

F-07
1. Q. same

Determine the bit rate and the resolution in the sampling of a seismic signal with dynamic range of 1 volt if the sampling rate is $F_s = 20$ samples/sec and we use an 8 bit A/D converter. What is the maximum frequency that can be present in the resulting digital seismic signal.

Soln.

$$\text{bit rate} \left(\frac{\text{bit}}{\text{second}} \right) = \frac{\text{bits}}{\text{Sample}} \times \frac{\text{Sample}}{\text{sec}}$$

$$= 8 \times 20 = 160 \text{ bps} \quad (\text{A})$$

$$\text{Resolution} = \frac{1 \text{ volt}}{2^b - 1} = \frac{1}{255} = 0.0039 \text{ V} = 3.9 \text{ mV} \quad (\text{B})$$

$$\text{Maximum} = \frac{F_s}{2} = \frac{20}{2} = 10 \text{ Hz} \quad (\text{C})$$

Note: $F_s = 20$ samples/sec $\Rightarrow T = \frac{1}{F_s} = \frac{1}{20} = 0.05 \text{ sec} = 50 \text{ ms}$

$$F_s = \frac{1}{T} = \frac{1}{50 \times 10^{-3}} = 20 \text{ samples/sec}$$

Sumon5-07

1(a) A continuous time sinusoid is always periodic regardless of the value of its frequency Ω . But a discrete time sinusoid $\cos \omega n$ is periodic only if ω is 2π times some rational number - justify this statement.

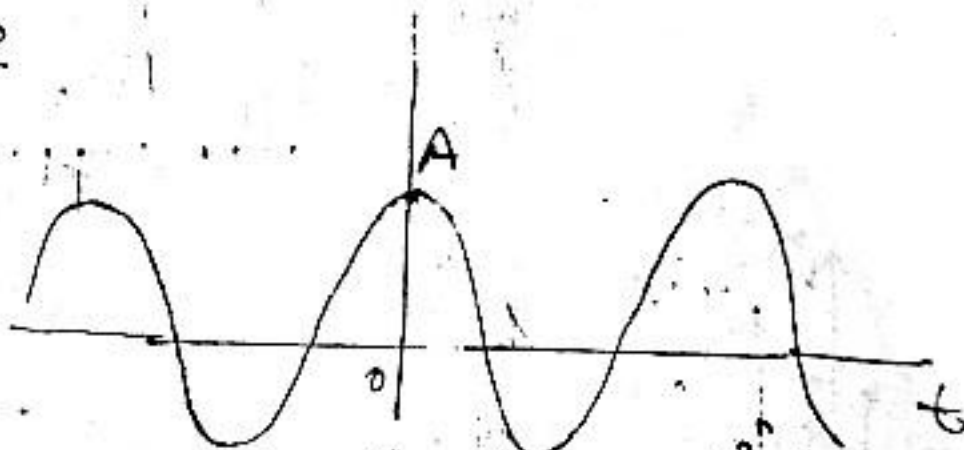
06 5-06

1(a) show that the highest rate of oscillation in a discrete time sinusoid is attained when $\omega = \pi$ (or $\omega = -\pi$)

① Ans: characteristic of CTS.

$$X(t) = A \sin(\underbrace{\Omega t + \theta}_{\text{phase}}) \quad -\alpha < t < \alpha$$

But $\Omega = 2\pi f$



(1) $X(t)$ is periodic for every fixed value of frequency f .

$$X(t + T_p) = X(t) \quad \text{the } T_p = \text{Time period}$$

and minimum value of T_p is the fundamental period.

(12) Continuous time sinusoidal signal with distinct (different) frequencies are themselves distinct.

Q.11) Ans: Highest rate of oscillation occurs at $\omega = \pi$ (or $f = \frac{1}{2}$ or $-\frac{1}{2}$) or $\omega = -\pi$

Ans: The sinusoidal signal sequence

$$x(n) = \cos \omega_0 n$$

when the frequency varies from 0 to π , we take the value $\omega_0 = 0, \frac{\pi}{8}, \frac{\pi}{4}, \frac{\pi}{2}, \pi$ and $f = 0, \frac{1}{16}, \frac{1}{8}, \frac{1}{4}, \frac{1}{2}$ mid period $N = 16, 8, 4, 2$ as depicted in Fig-1

So, period decreases and frequency increases. we can see the rate of oscillation increases as the frequency increase.

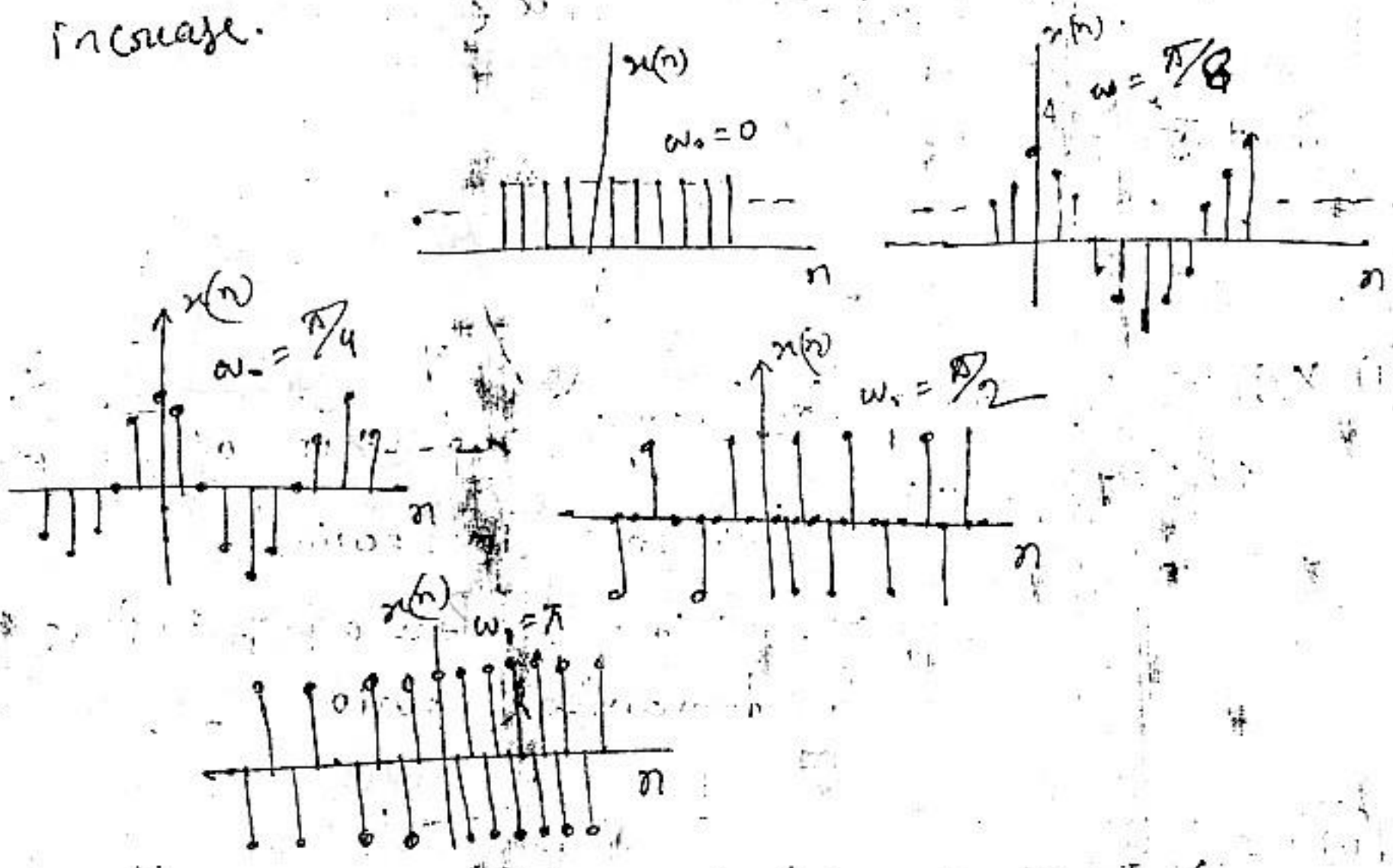


Fig: $x(n) = \cos \omega_0 n$ for various values of the frequency ω_0

D.T.S. is periodic only if its frequency is a rational no.
 Discrete time sinusoidal signal may be write

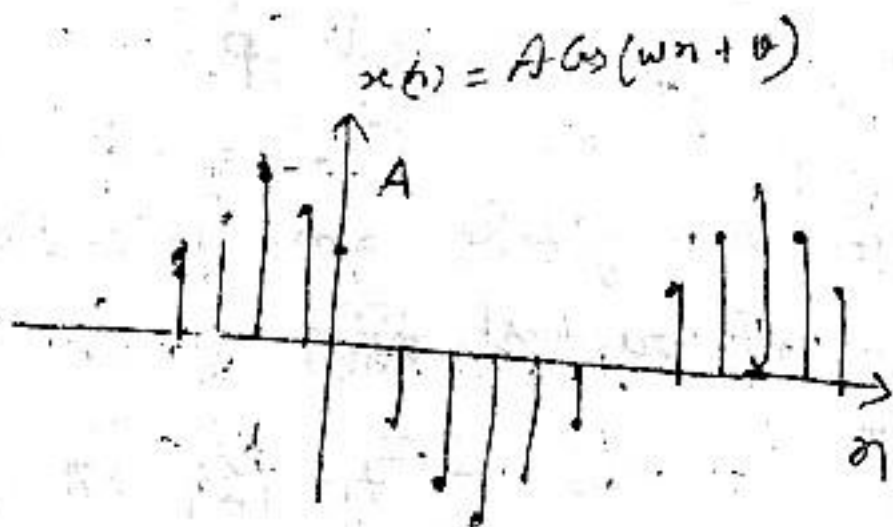
$$x(n) = A \cos(\omega n + \theta) \quad -\alpha < n < \alpha$$

But $\omega = 2\pi f$

$$x(n) = A \cos(2\pi f n + \theta) \quad -\alpha < n < \alpha$$

By D.T.S. $x(n)$ is periodic with period N ($N > 0$) if and only if

$$x(n+N) = x(n) \quad \text{for all } n$$



$$x(n) = A \cos(2\pi f n + \theta) \quad N - \text{fundn.}$$

$$\begin{aligned} x(n+N) &= A \cos[2\pi f(n+N) + \theta] \\ &= A \cos(2\pi f n + 2\pi f N + \theta) \end{aligned}$$

if $2\pi f N = 2\pi k$ k is the integer number.

$$f = \frac{k}{2N}$$

Example: $f_1 = \frac{31}{60}$ fundamental period $N_1 = 60$

$$f_2 = \frac{30}{60} = \frac{1}{2}, \quad N_2 = 2$$

So, (k or value n vary \Rightarrow fundamental frequency f vary also)

5-07
 (b) state the advantage of digital over analog signal

As: (1) Reconfiguration

2. Accuracy

3. Storage

4. Implementation of sophisticated Algorithm

5. cheap.

5-07
 (c) A digital communication link carries binary-coded words representing samples of an input signal

$$x_a(t) = 7 \cos(2400\pi t) + 3 \sin(2600\pi t + \pi/2)$$

The link is operated at 20,000 bits/sec and each input sample is quantized into 1024 different voltage levels.

The quantizer covers a range of 10V. Find.

(i) Nyquist rate, (ii) Folding frequency (F_s)

(iii) Discrete time signal $x(n)$

(iv) analog signal $y_a(t)$

(v) Resolution Δ

(vi) average power,

(vii) mean-square error power P_e

(viii) SQNR

(ix) SQNR

SumonSolⁿ:

Given,

$$L = 1024$$

$$x_a(t) = 7 \cos(1400\pi t) + 3 \sin(2600\pi t + \pi/2)$$

$$= 7 \cos(1400\pi t) + 3 \left\{ \sin 2600\pi t \cos \frac{\pi}{2} + \cos 2600\pi t \sin \frac{\pi}{2} \right\}$$

$$x_a(t) = 7 \cos(1400\pi t) + 3 \cos(2600\pi t) \quad \text{--- (1)}$$

$$(a) \quad F_1 = \frac{1400}{2} = 700 \text{ Hz} \quad F_2 = \frac{2600}{2} = 1300 \text{ Hz}$$

Thus $F_{\max} = 1300 \text{ Hz}$

$$F_s > 2F_{\max} = 2600 \text{ Hz}$$

The Nyquist Rate is $F_N = 2F_{\max} = 2600 \text{ Hz}$

(b) we know,

$$L = 2^b$$

$$1024 = 2^b$$

$$\log_{10} 1024 = \log_{10} (2^b)$$

$$b = \frac{\log_{10} 1024}{\log_{10} 2} = 10 \text{ bit/sample}$$

we know,

$$\frac{\text{bit}}{\text{sec}} \quad F_s = \frac{\text{Sample}}{\text{sec}}$$

$$\frac{\text{bit/sec}}{\text{bit/sample}} = \frac{20,000}{10} = 2000$$

$$\therefore F_s = 2000 \text{ Hz}$$

$$\therefore \text{Folding frequency} = \frac{F_s}{2} = \frac{2000}{2} = 1000 \text{ Hz}$$

(c) $F_s = 2 \text{ kHz}$

We know, discrete time signal,

$$x(n) = x_a(nT) = x_a\left(\frac{n}{F_s}\right)$$

$$\begin{aligned} x(n) &= 7 \cos 2\pi \left(\frac{7}{2}\right)n + 3 \cos 2\pi \left(\frac{13}{2}\right)n \\ &= 7 \cos 2\pi \left(3 + \frac{1}{2}\right)n + 3 \cos 2\pi \left(6 + \frac{1}{2}\right)n \\ &= 7 \cos \pi n + 3 \cos \pi n \end{aligned}$$

$$x(n) = 10 \cos \pi n$$

(d) ~~$y_a(t) = 10 \cos 1000\pi t$~~
Analog time signal,
 $y_a(t) = 10 \cos \pi F t$

$f = 1 \text{ Hz}$

B.V

$$\begin{aligned} F &= f F_s \\ &= 2000 \text{ Hz} \end{aligned}$$

$$y_a(t) = 10 \cos 2000\pi t$$

(e) Resolution $\Delta = \frac{2A}{2^b - 1}$
Analog processor
 $\frac{10V}{2^b - 1} = 0.000977$

$$= \frac{x_{\max} - x_{\min}}{2^b - 1}$$

$$= \frac{1300 - 700}{2^{10} - 1}$$

$$= \frac{60.0}{1023}$$

$$= 0.586$$

(f) Average power

$$P_{av} = \frac{A^2}{2} = \frac{(10)^2}{2} = 50 \text{ W}$$

Mean square error power,

$$(g) P_e = \frac{A^2/3}{2^{2b}} = \frac{(10)^2/3}{2^{20}} = \frac{(10)^2}{2^{20} \times 3} = \frac{100}{3 \times 2^{20}} = 3.17 \times 10^{-5}$$

$$(h) \text{SQNR/SNR} = \frac{P_n}{P_L} = \frac{3}{2} = 2^{2b}$$

$$= 1.572 \times 10^6 \text{ (Ans)}$$

$$(i) \text{SQNR (dB)} = 10 \log_{10}(\text{SQNR})$$

$$= 71.97 \text{ (dB)} = 61.97 \text{ (Ans)}$$

5-06
4(c)

Briefly describe the interpolation methods used in D/A converter.

Ans

Interpolation as shown in Fig (k) to connect successive samples with straight-line segments.

Better interpolation can be achieved by using more sophisticated higher order interpolation techniques.

In general, suboptimum interpolation techniques result in passing frequencies above the folding frequency. Such frequency components are undesirable and are usually removed by passing the output of the interpolator through a proper analog filter, which is called a postfilter or smoothing filter.

