* Solve of math for Power system Stability

F-06 **

F-07

8(b) Q: A 50Hz generator is suppling 80% of Pmax to an infinite bus through a reactive network. A fault occurs which increases the reactance of the network between the generator internal voltage and the infinite bus by 300%. When the fault is cleared the maximum power that can be delivered is 85% of the original max^m power (i.e. Pmax). Determine the critical clearing angle for this condition.

Solution:-

Now,

$$P_e = 0.8\, P_{max}$$

$$= P_{max} \sin \delta_0$$

$\therefore$ ~~$\delta_0 = 53°$~~ $\therefore \delta_0 = \sin^{-1}(0.8)$

$= 53.13° = 0.92$ rad.

Prefault, $P_{max} = \dfrac{V_1 V_2}{X}$ --- (i)

During fault,

$$\boxed{r_1 P_{max} X_f = \frac{V_1 V_2}{X}}$$

$\Rightarrow r_1 P_{max} = \dfrac{V_1 V_2}{3X}$ $\quad | \because X_f = 3X$

--- (ii)

(i) ÷ (ii) $\Rightarrow \dfrac{1}{r_1} = 3$

$\Rightarrow r_1 = 0.33$

after fault clearance, ~~$r_2\, max$~~

$$r_2 P_{max} = 0.85\, P_{max}$$

$$\Rightarrow r_2 = 0.85$$

$$\boxed{\delta_m = \pi - \sin^{-1}\left(\frac{\sin\delta_0}{r_2}\right)}$$

$$= \pi - \sin^{-1}\left(\frac{0.8}{0.85}\right)$$

$$= 180^\circ - 70.25^\circ$$

$$= 109.74^\circ$$

$$= \frac{109.74}{57.3}$$

$$\simeq 1.92 \text{ Radian.}$$

Now critical clearing angle,

$$\boxed{\delta_{cr} = \cos^{-1}\left[\frac{(\delta_m - \delta_0)\sin\delta_0 - r_1\cos\delta_0 + r_2\cos\delta_m}{r_2 - r_1}\right]}$$

$$= \cos^{-1}\left[\cancel{1.92 - 0.9}\right]$$

$$= \cos^{-1}\left[\frac{(1.92 - 0.92)\sin 53.13^\circ - .3\cos 53.13^\circ + .85\cos 109.74^\circ}{0.85 - 0.33}\right]$$

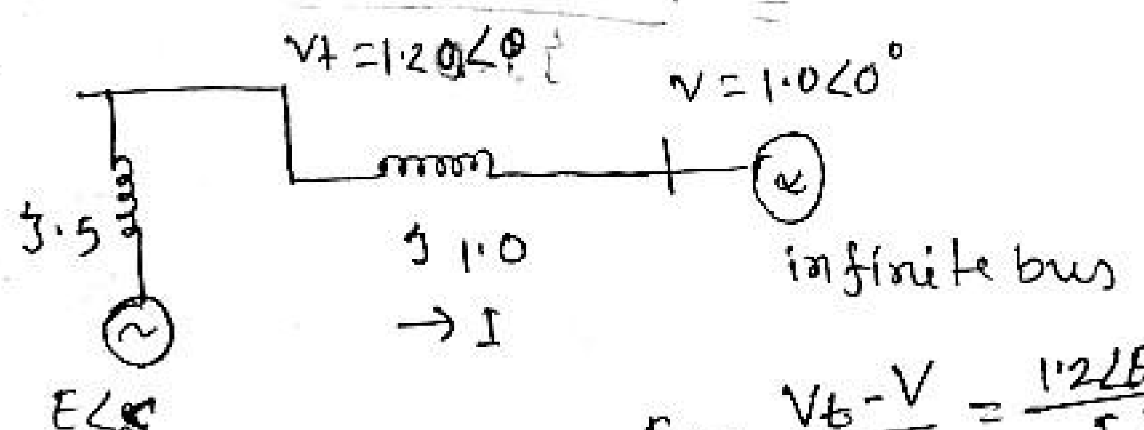
$$= \cos^{-1}\left[\frac{.799 - 0.18 - 0.287}{0.85 - 0.33}\right]$$

$$= 50.3^\circ \text{ Ans.}$$

F-07

7(b) Q: Find the steady state power limit of a system consisting of a generator equivalent reactance of 0.50 PU connected to an infinite bus through a series reactances of 1.0 P.U. The terminal voltage of the generator is held at 1.20 PU and the voltage of the infinite bus is 1.0 P.U.

Solution:-



$$I = \frac{V_t - V}{jx} = \frac{1.2\angle\theta - 1.0}{j1.0}$$

Now, $E = V_t + j0.5 \times I$

$$= 1.2\angle\theta + j.5 \times \left(\frac{1.2\angle\theta - 1.0\angle 0^\circ}{j1.0}\right)$$

$$= 1.8\angle\theta - 0.5$$

$$= 1.8(\cos\theta + j\sin\theta) - 0.5$$

$$= (1.8\cos\theta - 0.5) + j1.8\sin\theta.$$

The max^m power limit occurs when δ becomes 90°.

$$1.8\cos\theta - 0.5 = 0$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{0.5}{1.8}\right)$$

$$\therefore \theta = 73.87^\circ$$

so, ~~E = (1.8 0.~~

$$E = (1.8\cos\theta - 0.5) + j1.8\sin\theta$$

$$= 1.8\cos 73.87^\circ - 0.5 + j\,1.8\sin 73.87^\circ$$

$$= 1.729\angle 90^\circ$$

Now, $P_{max} = \frac{|E||V|}{X_{total}}$

$= \frac{1.729 \times 1.0}{(0.5+1.0)}$

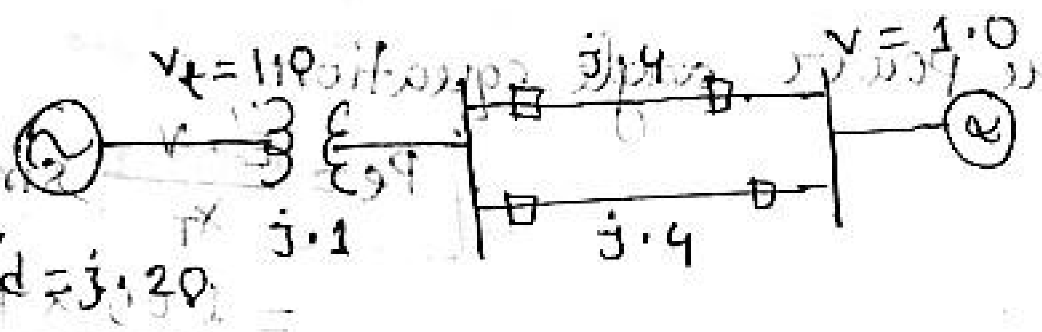
$= 1.152$ P.U.

$I = \frac{V_t - V}{j1.0}$

$= \frac{1.2\angle\theta - 1.0\angle 0^\circ}{j1.0}$

SP-07

8(c) The reactances of all the elements of a power system. show by the one line diagram, are given in the figure on a common base. The machine is delivering 1.0 PU power to an infinite bus. Both the generator terminal voltage and the infinite bus voltage are 1.0 P.U. Determine the power angle equation for the system applicable during a 3-phase fault on the middle of one of the lines.



Solution:- The series reactance between the terminal and the infinite bus is,

$$X = 0.1 + \frac{0.4}{2} = 0.3 \text{ P.U.}$$

terminal voltage $V_t = 1.0\angle\alpha$

infinite bus voltage $V = 1.0\angle 0^\circ$

$\therefore$ According to this problem $\boxed{\frac{V_t \cdot V}{X}\sin(\alpha - 0^\circ) = 1.0}$

$$\Rightarrow \frac{1.0 \times 1.0}{.3}\sin\alpha = 1$$

$$\therefore \alpha = 17.458^\circ$$

so terminal voltage $V_t = 1.0\angle 17.458^\circ$
$= 0.954 + j0.3$

The output current for generator is,

$$I = \frac{V_t - V}{X} = \frac{1.0\angle 17.458 - 1.0\angle 0^\circ}{j0.3}$$

$1.012\angle 8.72^\circ$

The transiont instand voltage of the machine is,

$$\boxed{E' = V_t + I \cdot jx'_d}$$

$$= (0.954 + j0.3) + (1.0 + j.1535)\, j0.2$$

$$= 0.923 + j0.5$$

$$= 1.05 \angle 28.44^{\circ}$$

The total series reactance between the generater and infinite buss.

$$X_T = 0.2 + 0.1 + \frac{0.4}{2}$$

$$= 0.5$$

The power angle equation,

$$\boxed{P_e = \frac{E' \cdot V}{X_T} \sin\delta}$$

$$= \frac{1.05 \times 1.0}{0.5} \sin\delta$$

$$= 2.1 \sin\delta$$

The normal operating condition, the swing equation,

$$\frac{H}{180f} \frac{d^2\delta}{dt^2} = 1.0 - 2.1 \sin\delta \text{ p.u}$$

SP-06

7(c) A synchronous generator of reactance 1.20 P.U is connected to an infinite busbar $|V| = 1.0$ P.U. through transformers and a line of total reactance of 0.60 PU. The generator no load voltage is 1.20 P.U and $H = 4$ MW-S/MVA. The resistance and machine damping ~~may be~~ is negligible. The system frequency is 50 Hz. Calculate the frequency of natural oscillations if the generator is loaded to

i) 50% and (ii) 80% of its maximum power limit.

Ans:- (i) 50% loading,

Here $P_e = 0.5 P_{max}$

We know, $P_e = P_{max} \sin\delta$

$\Rightarrow 0.5 P_{max} = P_{max} \sin\delta$

$\Rightarrow \sin\delta_0 = 0.5$

$\therefore \delta_0 = 30°$

Now, $$P_s = \frac{dP_e}{d\delta}\Big|_{\delta_0} = \frac{d}{d\delta}\left(\frac{|E||V|}{X}\sin\delta_0\right)$$

$$= \frac{|E||V|}{X}\cos\delta_0$$

$$= \frac{1.2 \times 1.0}{(1.2+0.6)}\cos 30°$$

$= 0.577$ MW(PU)/elec. rad

∴ The characteristic equation is,

$$\boxed{S^2 = -\frac{\pi f_0}{H} P_S}$$

$$= -\frac{50\pi}{4} \times 0.577$$

$$\therefore S = \pm j\sqrt{\frac{50\pi \times 0.577}{4}} = \pm j\,4.76$$

∴ Frequency of oscillation $= 4.76$ rad/s

$$= \frac{4.76}{2\pi} \text{ c/s}$$

$$= 0.758 \text{ c/s (Hz)}.$$

(ii) For 80% loading, $0.8 P_{max} = P_e$

$$\sin\delta_0 = \frac{P_e}{P_{max}}$$

$$= \frac{0.8 P_{max}}{P_{max}} = 0.8$$

$$\therefore \delta_0 = 53.1^\circ$$

$$\therefore P_s = \frac{d}{d\delta}(P_{max}\sin\delta)\Big|_{\delta_0} = \frac{|E||V|}{X}\cos\delta_0$$

$$= \frac{1.2 \times 1}{1.8}\cos 53.1^\circ$$

$$= 0.4 \text{ MW (p.u)/elec rad.}$$

$$S = \pm j\sqrt{\frac{50\pi \times 0.4}{4}}$$

$$= \pm j\,3.96$$

∴ The frequency of oscillation $(\omega_n) = 3.96$

$$= \frac{3.96}{2\pi} \text{ Hz}$$

$$= 0.63 \text{ Hz}$$

8(a) A 60 Hz generator having H = 6.0 MJ/MVA is delivering real power of 1.0 P.U. to an infinite bus through a purely reactive network when the occurrance of a 3 phase fault reduces the generator output power to zero. The maximum power that could delivered is 2.5 P.U. When the fault is cleared, the original network condition is again restored and Determine the critical clearing angle and the critical clearing time.

Ans:— Under normal operating condition

$$P_m = P_e = 1.0 \text{ P.U.}$$

$$P_{max} = 2.5 \text{ P.U.}$$

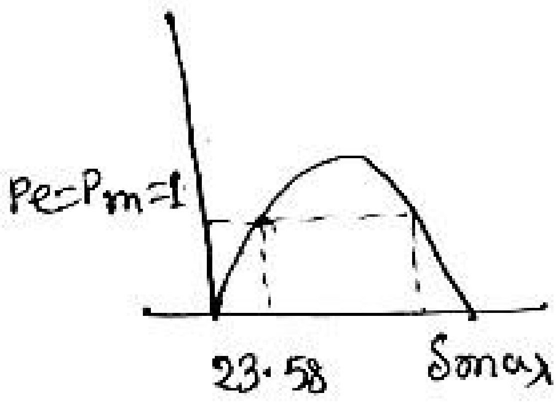
the power angle equation,

$$P = P_{max} \sin \delta_0$$

$$\therefore 1 = 2.5 \sin \delta_0$$

$$\Rightarrow \delta_0 = \sin^{-1}(1/2.5)$$

$$= 25.58.$$



Critical clearing angle

$$\delta_{cr} = \cos^{-1}\left[-\cos\delta_0 + (\pi - 2\delta_0)\sin\delta_0\right]$$

$$= \cos^{-1}\left[-\cos 23.58^\circ + \left(3.14 - \frac{2 \times 23.58}{180}\right) \sin 23.58^\circ\right]$$

$$\Rightarrow \delta_{cr} = \cos^{-1}\left[-.9165 + 2.3184 \times 0.4\right]$$

$$= \cos^{-1}\left[0.0109657\right] = 89.37^\circ = 1.5598 \text{ rad.}$$

Critical clearing time

$$t_{cr} = \sqrt{\frac{4H(\delta_{cr} - \delta_0)}{\omega_s \cdot P_m}}$$

$$= \sqrt{\frac{4 \times 6 (89.37 - 23.58)}{2 \times 180 \times 60 \times 1}}$$

$$= \sqrt{\frac{4 \times 6 \times 65.79}{360 \times 60}} = \sqrt{0.0731}$$

$= 0.27$ sec.

P-06

3φ complex power interms of symmetrical components :—

$S = V_a I_a^* + V_b I_b^* + V_c I_c^*$, | we know, $V_P = A V_S$

$= \begin{bmatrix} V_a & V_b & V_c \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}^*$

$= \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}^T \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}^*$

$= \begin{bmatrix} A V_s \end{bmatrix}^T \begin{bmatrix} A I_s \end{bmatrix}^*$

$= V_s^T A^T \cdot A^* \cdot I_s^*$

Now, $A^T = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}$.

$A^* = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^* & a^2 \\ 1 & a^2 & a^* \end{bmatrix}$

Let. $a = 1\angle 120^\circ$

$a^* = 1\angle -120^\circ = 1\angle +240^\circ = a^2$

$a^{2*} = 1\angle 240^\circ = 1\angle 120^\circ = a$

$\therefore S = \begin{bmatrix} V_{a0} & V_{a1} & V_{a2} \end{bmatrix} \times 3 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}^*$

$= 3V_{a1} I_{a1}^* + 3V_{a2} I_{a2}^* + 3 I_{a0}^* V_{a0}$